Lecture 10

Median Filtering, Mode Filtering, and Rank Leveling

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Median Filtering

- We have seen that smoothing (low pass) filters reduce noise.
- However, the underlying assumption is that the neighboring pixels represent additional samples of the same value as the reference pixel, i.e. they represent the same feature.
- At edges, this is clearly not true, and blurring of features results.
Median Filtering

- You have used convolution techniques to implement weighting kernels as a neighborhood function, which represented a linear process.
- There are also nonlinear neighborhood operations that can be performed for the purpose of noise reduction that can do a better job of preserving edges than simple smoothing filters.

Median Filtering

- One such method is known as **median filtering**.
- In median filtering, the neighboring pixels are ranked according to brightness (intensity) and the median value becomes the new value for the central pixel.
- Median filters can do an excellent job of rejecting certain types of noise, in particular, “shot” or impulse noise in which some individual pixels have extreme values.
- In the median filtering operation, the pixel values in the neighborhood window are ranked according to intensity, and the middle value (the median) becomes the output value for the pixel under evaluation.
Median Filtering

- In particular, compared to the smoothing filters examined thus far, median filters offer three advantages:
  - No reduction in contrast across steps, since output values available consist only of those present in the neighborhood (no averages).
  - Median filtering does not shift boundaries, as can happen with conventional smoothing filters (a contrast dependent problem).
  - Since the median is less sensitive than the mean to extreme values (outliers), those extreme values are more effectively removed.
Median Filtering

• The median is, in a sense, a more robust “average” than the mean, as it is not affected by outliers (extreme values).
• Since the output pixel value is one of the neighboring values, new “unrealistic” values are not created near edges.
• Since edges are minimally degraded, median filters can be applied repeatedly, if necessary.

Median Filtering

• Considerations:
  – The median filter is more expensive to compute than a smoothing filter. Clever algorithms can save time by making use of repeating values as the neighborhood window is slid across the image.
  – Median filters are nonlinear:
    Median[A(x)+B(x)] ≠ median[A(x)]+median[B(x)]
    This must be taken into account if you plan on summing filtered images.
Hybrid Median Filtering

- Median filters can tend to erase lines narrower than ½ the width of the neighborhood. They can also round off corners.
- Hybrid median filters can get around these problems.
- The hybrid median filter is a three step ranking process that uses two subgroups of a 5x5 neighborhood.
- These subgroups are drawn from pixels parallel to the image frame edges, and at 45° to the edges, centered on the reference pixel.
Median Filtering

• The median for each subgroup is determined.
• These two values are then compared to the original pixel value, and the median for these three values becomes the output value for the pixel in the filtered image.
• Larger neighborhoods permit the defining of additional subgroup orientations.
Mode Filters

• A technique related to median filtering is mode filtering.
• Mode filtering accomplishes noise reduction, and also can provide for edge enhancement.
• The mode of a distribution is defined as the most likely value.
• However, for small neighborhoods, the mode is poorly defined, so a truncated median filter can be used to approximate the mode.
Mode Filtering

• In an asymmetric distribution (as would be the case with a neighborhood near and edge) the mode is the highest point in the neighborhood histogram and the median is closer to the mode than to the mean value.

• In the truncated median method, values farthest from the mean are discarded so that the median is shifted towards the mode.

• The new median becomes the output value for the central pixel.
Mode Filtering

- One consequence of repeated application of the mode filter is that the number of distinct intensity values is reduced (the so-called “posterization” effect).
- This is also what would happen if the bit depth of the image were reduced.
Rank Leveling

• **Rank leveling** is a useful technique in circumstances where the background intensity varies widely and the features of interest are limited in size and are smaller than the scale of the background variations.

• Contrast between the features of interest and background is assumed.

Rank Leveling

• In rank leveling a 3x3 or 5x5 neighborhood is typically used.

• As before, the center pixel in the neighborhood is evaluated.

• If the background is lower in intensity than the feature, the lowest pixel intensity in the neighborhood becomes the new center pixel value in the output image.
Rank Leveling

• If the background is brighter, then the highest pixel intensity is used.
• This process is sometimes referred to as **gray-scale erosion or dilation**.
• The result of this process is that the size of the features is reduced by the size of the neighborhood radius along with extending the local intensity values into the area previously covered by the features.
Rank Leveling

- This process can be continued until the features of interest are eliminated/separated altogether.
- The number of repetitions required to remove features is based on the relative size of the neighborhood to the features, e.g. a feature less than or equal to four times the width of the neighborhood will be removed in four iterations.

In the absence of precise feature dimensions, the histogram can be used with each iteration to determine when the features have been removed.
- The result is an image with the large scale variations of the original background present, but no features.
- Subtraction of the leveled image from the original produces an image with clearly defined features which can be extracted by thresholding.
Rank Leveling

- The subtracted image may be processed by any of a number of mapping methods (e.g. polynomial expansion) to increase the contrast of the features, allowing for the discrimination of very small features.
- This is important in histological analysis of tissue samples.
MATLAB Syntax

- Median filtering is accomplished using the `medfilt2` function:

  \[ B = \text{medfilt2}(A, [m \ n]); \]

  Where \( B \) is the output image, \( A \) is the input image, and \( m \times n \) is the size of the neighborhood to be used for median determination.

- If \( m=n \) the neighborhood will be square.
MATLAB Syntax

- The `ordfilt2` function allows additional flexibility in implementing neighborhood rank filtering functions. The syntax is:
  
  \[ B = \text{ordfilt2}(A, \text{order}, \text{domain}); \]

- \( B \) is the output image, \( A \) is the input image, \( \text{domain} \) specifies the neighborhood, and \( \text{order} \) represents the rank within the neighborhood intensity values to be used as the output pixel value.

- For example:
  
  \[ B = \text{ordfilt2}(A, 5, \text{ones}[3 \ 3]); \]

  implements a median filter using a 3x3 neighborhood.

- In the same case, if you wanted to use the maximum of the neighborhood:
  
  \[ B = \text{ordfilt2}(A, 9, \text{ones}[3 \ 3]); \]
MATLAB Syntax

- As you would expect, the corresponding minimum filter would be:
  
  \[ B = \text{ordfilt2}(A,1,\text{ones}[3\ 3]); \]
- Remember that for the order index, 1 represents the minimum value.
- Non-square neighborhoods may also be specified.

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MATLAB Syntax

- In the neighborhood (domain) specification the array is used as a binary table.
- That is, array elements with value = 1 are used in defining the neighborhood, and array elements with value = 0 are ignored.
- Thus it is possible to define neighborhoods like those discussed for hybrid median filtering.
MATLAB Syntax

- In the examples just shown, the `ones` function was used to define a square neighborhood in which all of the values were to be used in the ranking process.
- To produce neighborhoods consisting of a subset of a square matrix, simply set the unwanted array values to zero.

```matlab
MATLAB Syntax

- If you want to produce a maximum filter with a neighborhood using only pixels that are not diagonals of a 3x3 neighborhood:
  \[ B = \text{ordfilt2}(A,4,[0 \ 1 \ 0;1 \ 0 \ 1;0 \ 1 \ 0]); \]
- The corresponding minimum filter would be:
  \[ B = \text{ordfilt2}(A,1,[0 \ 1 \ 0;1 \ 0 \ 1;0 \ 1 \ 0]); \]
```
MATLAB Syntax

• For more elaborate configurations based on larger arrays, it is possible to define a named binary array and call it by name rather than specifying it explicitly in the `ordfilt2` function.

• So if you wanted to use a neighborhood using only the diagonals of a 5x5 square for a minimum filter, define the array (C) and use:

\[
B = \text{ordfilt2}(A,1,C);
\]

MATLAB Syntax

• Where:

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]